Math 2A Quiz 1 Version 1
Fri Jan 15, 2016

NAME YOU ASKED TO BE CALLED IN CLASS: ______

SCORE: _____ / 30 POINTS

1. **NO CALCULATORS OR NOTES ALLOWED**
2. **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS**
3. **SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS**

Consider the IVP $y' = 2y^2 - 3x$, $y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$. **SCORE: 4 / 4 PTS**

$\frac{dy}{dx} = 2y^2 - 3x$

$y(1.2) = y(1) + y' \cdot \Delta h$

$= -2 + (2\times 4 - 3)\times 0.2$ 1

$= -2 + 1 = -1$ 1

$y(1.4) = y(1.2) + y' \Delta h$

$= -1 + (2\times 1 - 3\times 1.2)\times 0.2$

$= -1 + (-1.6)\times 0.2$ 1

$= -1 - 0.32 = -1.32$ 1

Determine if $y = A\sqrt{x} + \dfrac{B}{x^2} + \dfrac{x^2}{4}$ is a family of solutions of the DE $4x^2y'' + 10xy' - 4y = 5x^2$. **SCORE: 6 / 6 PTS**

State your conclusion clearly.

$y = Ax^{\frac{1}{2}} + B\cdot x^{-2} + \frac{1}{4}x^2$

$y' = \frac{1}{2}Ax^{-\frac{1}{2}} - 2Bx^{-3} + \frac{1}{2}x$ 1

$y'' = -\frac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \frac{1}{2}$ 1

$4x^2(-\frac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \frac{1}{2}) + 10x(\frac{1}{2}Ax^{-\frac{1}{2}} - 2Bx^{-3} + \frac{1}{2}x) - 4(Ax^{\frac{1}{2}} + Bx^{-2} + \frac{1}{4}x^2)$ 1

$= -Ax^{\frac{1}{2}} + 24Bx^{-2} + 2x^2 + 5Ax^{\frac{1}{2}} - 20Bx^{-2} + 5x^2 - 4Ax^{\frac{1}{2}} - 4Bx^{-2} - x^2$

$= 7x^2 - x^2 = \boxed{6x^2 \neq 5x^2}$ 2

SO IT IS NOT A FAMILY OF SOLUTIONS OF THE D.E 1

In certain population models, a group will go extinct if and only if its population is below a certain level (called the survival threshold P_S). Write a differential equation for the population of a group which is going extinct, if the rate of change of its population is proportional to the difference between the threshold and the existing population. **Justify your answer properly, but briefly.** SCORE: ____ / 4 PTS

NOTE: The signs of all constants should be stated clearly.

✱ $\frac{dP}{dt} = k \cdot (P_S - P)$

Since a group will go extinct if and only if P is below P_s

so $P_s - P > 0$, and since the grap is going extinct, $\frac{dP}{dt} < 0$. $\frac{1}{2}$

✱ so $\boxed{k < 0}$ in this case

✱

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $\frac{dy}{dx} = \frac{\sqrt[3]{y-2}}{x+6}$, $y(8) = 2$? **Justify your answer properly, but briefly.** SCORE: 0 / 4 PTS

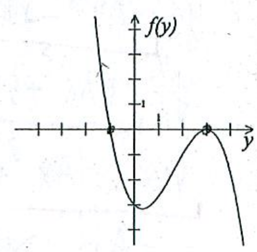
THAT THE FIRST ORDER DERIVATIVE DOES NOT EXIST AT $x = -6$,

AND AT $y(8)$ $\frac{dy}{dx} = \frac{\sqrt[3]{2-2}}{8+6} = 0$ IS A CRITICAL POINT OF y.

THAT THE VALUE OF y IS ALWAYS GREATER THAN TWO

THAT THERE EXIST A UNIQUE SOLUTION WHICH SATISFIES THE IVP $\frac{dy}{dx} = \frac{\sqrt[3]{y-2}}{x+6}$; $y(8) = 2$

Consider the autonomous DE $\frac{dy}{dx} = f(y)$, where $f(y)$ is the function whose graph is shown on the right. SCORE: ____ / 6 PTS



[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
You must draw a phase portrait to get full credit.

$f(y) < 0$, $\frac{dy}{dx} < 0$

3

$f(y) < 0$, $\frac{dy}{dx} < 0$

-1

$f(y) > 0$. $\frac{dy}{dx} > 0$

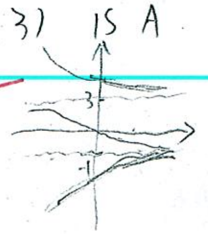
2

✱ 1

$f(-1)$ IS A STABLE EQUILIBRIUM SOLUTION

$f(3)$ IS A SEMI-STABLE EQUILIBRIUM SOLUTION

1



[b] If $y = m(x)$ is a solution of the DE such that $m(4) = 2$, what is $\lim_{x \to \infty} m(x)$?

$\lim_{x \to \infty} m(x) = -1$. 1